Lecture 16

Extreme Value Theorem

Let R be a closed, bounded set in the plane. Also, let f be continuous on R. Then f has both a max & a min value on R.

Finding Extreme Values in a Region

1. Find the critical points and evaluate f at those points.
2. Find the extreme values on the boundary of R.
3. Find the max & min of these values.

Ex. 1

Find the absolute max & min values of $f(x) = x^2 - 2xy + 2y$ on the region $R = \{(x,y) \mid 0 \le x \le 3,\ 0 \le y \le 2\}$

1. $f_x = 2x - 2y = 0$ $\qquad$ $f_y = -2x + 2 = 0$

$x = 1$

$2 - 2y = 0$

$y = 1$

critical point @ $(1,1)$

$f(1,1) = 1$

2. The region is a rectangle with 4 line segments

L_1: $0 \le x \le 3$, $y = 0$

$f(x,0) = x^2$ $\qquad$ $f(0,0) = 0$ $\qquad$ $f(3,0) = 9$

L_2: $0 \le x \le 3$, $y = 2$

$f(x,2) = x^2 - 4x + 4 = (x-2)^2$ $\qquad$ $f(0,2) = 4$ $\qquad$ $f(2,2) = 0$

L_3: $0 \le y \le 2$, $x = 0$

$f(0,y) = 2y$ $\qquad$ $f(0,0) = 0$ $\qquad$ $f(0,2) = 4$

L_4: $0 \le y \le 2$, $x = 3$

$f(3,y) = 9 - 6y + 2y = 9 - 4y$ $\qquad$ $f(3,0) = 9$ $\qquad$ $f(3,2) = 1$

3. The absolute max of f on R is $f(3,0) = 9$

The absolute min of f on R is $f(0,0) = f(2,2) = 0$

Ex. 2

Find the extreme values of $f(x,y) = 2x^2 - 3y^2$ on $x^2 + y^2 \le 4$

1. $f_x = 4x = 0$ $\qquad f_y = -6y = 0$

$x = 0$ $\qquad y = 0$

$f(0,0) = 0$

2. $x^2 + y^2 = 4$

$x^2 = 4 - y^2$

$g(y) = 2(4 - y^2) - 3y^2 = 8 - 5y^2$

$g_y = -10y = 0$

$y = 0$ $\qquad g(0) = 8 \rightarrow f(-2,0) = f(2,0) = 8$

$g(-2) = g(2) = -12 \rightarrow f(0,-2) = f(0,2) = -12$

3. The max of f on R is $f(-2,0) = f(2,0) = 8$

The min of f on R is $f(0,-2) = f(0,2) = -12$